Calculus AP® – Exam Preparation Questions

Multiple Choice What is

$$\lim_{h \to 0} \frac{\sin(\pi + h) - \sin \pi}{h}?$$

(A) -1 (B) 0

(C) 1

(D) The limit is nonexistent.

h	$\dfrac{\sin(\pi + h) - \sin \pi}{h}$
-0.1	
-0.01	
-0.001	
0.001	
0.01	
0.1	

Multiple Choice If $y = -5\sqrt[3]{15x^2 - 1}$, then $y' =$

(A) $\dfrac{10x}{(15x^2 - 1)^{2/3}}$. (B) $-\dfrac{5}{3(15x^2 - 1)^{2/3}}$.

(C) $-\dfrac{50x}{\sqrt[3]{15x^2 - 1}}$. (D) $-\dfrac{50x}{(15x^2 - 1)^{2/3}}$.

Newton's Method for Approximating the Zeros of a Function

Let $f(c) = 0$, where f is differentiable on an open interval containing c. Then, to approximate c, use these steps.

1. Make an initial estimate x_1 that is close to c. (A graph is helpful.)
2. Determine a new approximation

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

3. When $|x_n - x_{n+1}|$ is within the desired accuracy, let x_{n+1} serve as the final approximation. Otherwise, return to Step 2 and calculate a new approximation.

Each successive application of this procedure is called an **iteration.**

2.8 Newton's Method $f'(x_n) = 6x^2$ $f'(x_n) = -3e^{-3x} - 2x$

Example 1: Calculate four iterations of Newton's Method to approximate the zero of $f(x) = 2x^3 + 1$. Use $x_1 = -1$ as the initial guess.

n	x_n	$f(x_n)$	$f'(x_n)$	$\frac{f(x_n)}{f'(x_n)}$	$x_n - \frac{f(x_n)}{f'(x_n)}$
1					
2					
3					
4					

Example 2: Use Newton's Method to approximate the zero(s) of $f(x) = e^{-3x} - x^2$. Continue the iterations until two successive approximations differ by less than 0.0001.

n	x_n	$f(x_n)$	$f'(x_n)$	$\frac{f(x_n)}{f'(x_n)}$	$x_n - \frac{f(x_n)}{f'(x_n)}$
1					
2					

2.8 Newton's Method $f'(x_n) = \frac{1}{3}(x-4)^{-2/3}$

Example 3: The function $f(x) = (x-4)^{1/3}$ is not differentiable at $x = 4$. Show that Newton's Method fails to converge using $x_1 = 4.1$.

n	x_n	$f(x_n)$	$f'(x_n)$	$\frac{f(x_n)}{f'(x_n)}$	$x_n - \frac{f(x_n)}{f'(x_n)}$
1					
2					