2.7 Related Rates

Example 1:

If one side of a triangle, a, is increasing at a rate of 3 inches per minute while the other side, b, is decreasing at a rate of 3 inches per minute, which of the following must be true about the area A of the triangle?

(A) A is always increasing
(B) A is always decreasing
(C) A is decreasing only when a<b
(D) A is decreasing only when a>b
(E) A is constant

2.7 Related Rates

Guidelines for Solving Related-Rate Problems

1. Identify all *given* quantities and quantities *to be determined*. Make a sketch and label the quantities.
2. Write an equation involving the variables whose rates of change either are given or are to be determined.
3. Using the Chain Rule, implicitly differentiate both sides of the equation *with respect to time t*.
4. *After* completing Step 3, substitute into the resulting equation all known values for the variables and their rates of change. Then solve for the required rate of change.

Remark

When using these guidelines, be sure you perform Step 3 before Step 4. Substituting the known values of the variables before differentiating will produce an inappropriate derivative.

2.7 Related Rates

Verbal Statement	Mathematical Model
The velocity of a car after traveling for 1 hour is 50 miles per hour.	x = distance traveled $\frac{dx}{dt} = 50$ mi/h when $t = 1$
Water is being pumped into a swimming pool at a rate of 10 cubic meters per hour.	V = volume of water in pool $\frac{dV}{dt} = 10\ \text{m}^3/\text{h}$
A gear is revolving at a rate of 25 revolutions per minute (1 revolution = 2π rad).	θ = angle of revolution $\frac{d\theta}{dt} = 25(2\pi)$ rad/min
A population of bacteria is increasing at a rate of 2000 per hour.	x = number in population $\frac{dx}{dt} = 2000$ bacteria per hour

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<u>Example 2:</u> At noon, ship A is 150 km west of ship B. Ship A is sailing east at 35 km/h and ship B is sailing north at 25 km/h. At what rate is the distance between the ships changing at 4:00 PM?

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4. *Write an equation involving variables whose rates of change either are given or are to be determined.*

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3. Use the Chain Rule to differentiate both sides of the equation with respect to t.

4. Substitute the given information into the resulting equation and solve for the unknown rate.

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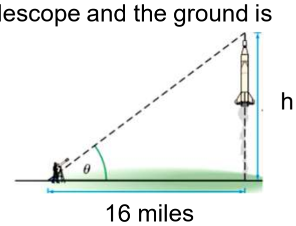
Example 3: A rocket travels vertically at a speed of 1200 mph. The rocket is tracked through a telescope by an observer located 16 miles from the launching pad. Find the rate at which the angle between the telescope and the ground is increasing 3 minutes after lift-off.



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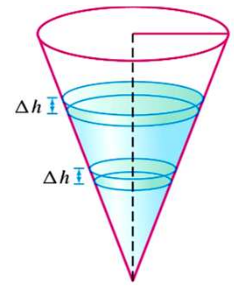
Example 4: Water pours into a conical tank of height 10 m and radius 4 m at a rate of $6m^3/\mathbf{min}$.

a. At what rate is the water level rising when the level is 5 m high?

b. As time passes, what happens to the rate at which the water level rises?



b. As time passes, what happens to the rate at which the water level rises?