## 5.1 Slope Fields and Euler's Method

- Use initial conditions to find particular solutions of differential equations.
- Use slope fields to approximate solutions of differential equations.
- Use Euler's method to approximate solutions of differential equations.

**Example 1**: Determine whether each function is a solution of the differential equation $y'' + y = 0$

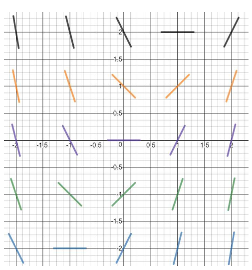
a. $y = \cos x$ b. $y = e^{-x}$ c. $y = C\sin x$

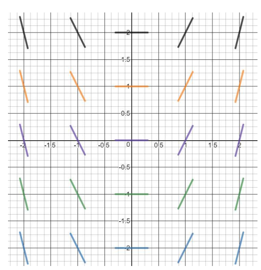
**Example 2**: For the differential equation $y' + 2xy = 0$, verify that $y = Ce^{-x^2}$ is a solution. Then find the particular solution determined by the initial condition $y = 2$ when $x = 0$

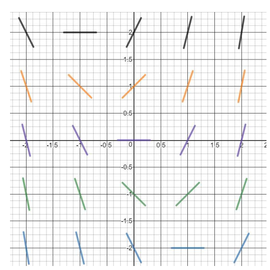
## 5.1 Slope Fields and Euler's Method

**Example 3**: Match each slope field with its differential equation.

*a.* $y' = 2x + y$ b. $y' = 2x - y$ c. $y' = 2x$

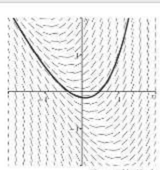






**Technology**

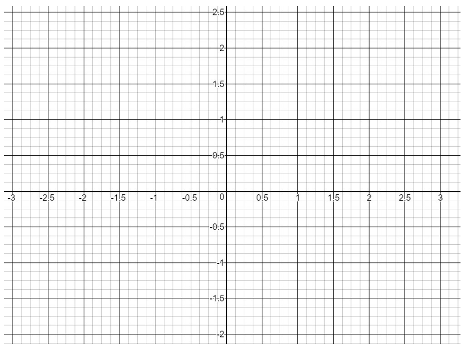
Drawing a slope field by hand is tedious. In practice, slope fields are usually drawn using a graphing utility. If you have access to a graphing utility that can graph slope fields, try graphing the slope field for the differential equation in Example 5. One example of a slope field drawn by a graphing utility is shown at the right.



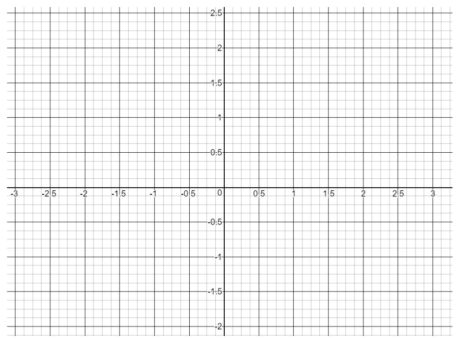
*Generated by Maple*

5.1 Slope Fields and Euler's Method

**Example 4**: Sketch the slope field for the differential equation $y' = 2 + x$ for the points (-2,2), (-1,2), and (0,2).



**Example 5**: Sketch a slope field for the differential equation $y' = y - 3x$. Use the slope field to sketch the solution that passes through the point (2,1).



5.1 Slope Fields and Euler's Method

**Example 6**: Use Euler's Method to approximate the particular solution of the differential equation $y' = y - x$ passing through the point (0,2). Use five steps of $h = 0.1$.

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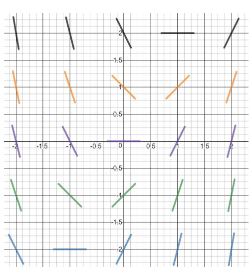

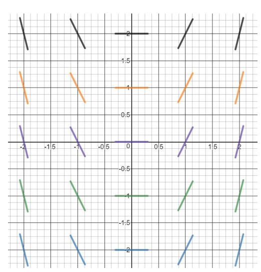

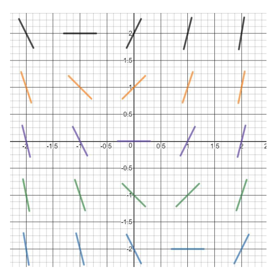

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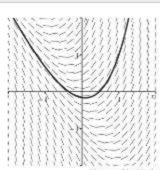

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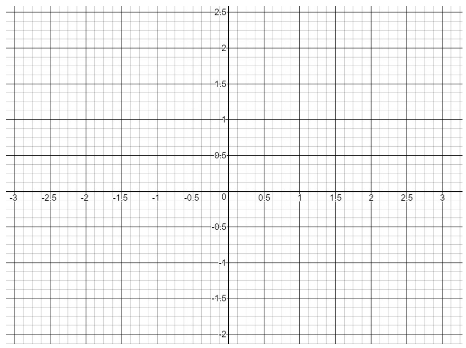


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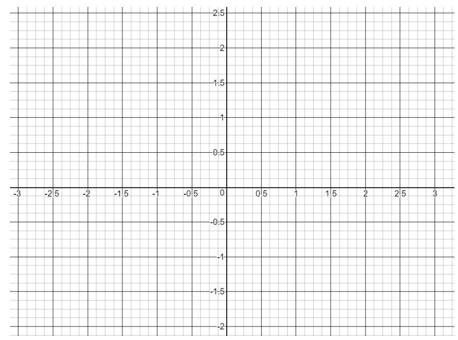


### 5.1 Slope Fields and Euler's Method

**Example 6**: Use Euler's Method to approximate the particular solution of the differential equation $y' = y - x$ passing through the point (0,2). Use five steps of $h = 0.1$.

5.3 Separation of Variables

- Recognize and solve differential equations that can be solved by separation of variables.
- Use differential equations to model and solve applied problems.

**Example 1**: Find the general solution of $(y\cos x)\frac{dy}{dx} = \sec x$.

**Example 2**: Given the initial condition $y(1) = 2$, find the particular solution of the equation $3x^2y^2y' - x = 0$.

5.3 Separation of Variables

**Example 3**: Find the equation of the curve that passes through the point $(3,2)$ and has a slope of $-\frac{4x}{y^3}$ at any point $(x, y)$.

**Example 4**: The rate of change in the number of foxes $N(t)$ in a population is directly proportional to $550 - N(t)$, where $t$ is the time in years. When $t = 0$, the population is 250, and when $t = 3$ the population has increased to 500. Find the population when $t = 5$.

5.3 Separation of Variables

**Example 5**: A new product is introduced through an advertising campaign to a population of 1.5 million potential customers. The rate at which the population hears about the product is assumed to be proportional to the number of people who are not yet aware of the product. By the end of 6 months, 375,000 people have heard of the product. How many will have heard of it by the end of 1 year?

5.3 Separation of Variables

**Example 6**: During a chemical reaction, substance A is converted into substance B at a rate that is proportional to the square of the amount of A. When $t = 0$, 80 grams of A is present, and after 1 hour ($t = 1$), only 25 grams of A remain. How much of A is present after 3 hours?

$L = population\ limit$

$$\frac{dy}{dt} = ky \ln \frac{L}{y}$$

**Example 7**: A population of 30 bobcats has been introduced into a national park. The forest service estimates that the maximum population the park can sustain is 250 bobcats. After 2 years, the population is estimated to be 45 bobcats. According to a Gompertz growth model, how many bobcats will there be 5 years after their introduction?