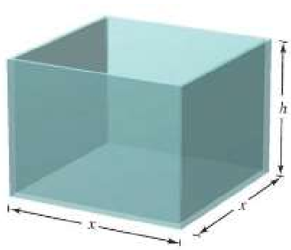
Guidelines for Solving Applied Minimum and Maximum Problems

1. Identify all *given* quantities and all quantities *to be determined.* If possible, make a sketch.
2. Write a **primary equation** for the quantity that is to be maximized or minimized. (A review of several useful formulas from geometry is presented inside the back cover.)
3. Reduce the primary equation to one having a *single independent variable.* This may involve the use of **secondary equations** relating the independent variables of the primary equation.
4. Determine the feasible domain of the primary equation. That is, determine the values for which the stated problem makes sense.
5. Determine the desired maximum or minimum value by the calculus techniques discussed in Sections 3.1 through 3.4.

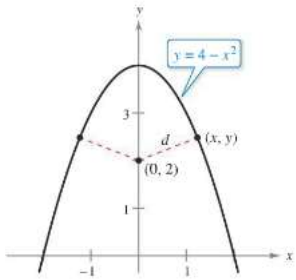
<u>Example 1:</u> A manufacturer wants to design an open box having a square base and a surface area of 108 square inches, as shown in the image. What dimensions will produce a box with maximum volume?



Open box with square base:

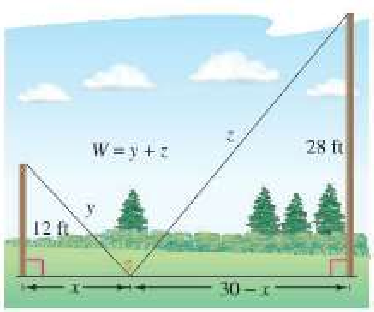
3.6 Optimization Problems

Example 2: Which points on the graph of $y = 4 - x^2$ are closest to the point $(0,2)$?



3.6 Optimization Problems

Example 3: Two posts, one 18 feet high and the other 32 feet high, stand 35 feet apart. They are to be stayed by two wires, attached to a single stake, running from ground level to the top of each post. Where should the stake be placed to use the least amount of wire?

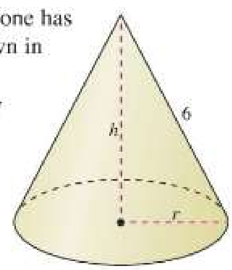


3.6 Optimization Problems

Example 4: The demand function for a product is modeled by $p = 56e^{-0.000012x}$, where p is the price per unit (in dollars) and x is the number of units. What price will yield a maximum revenue?

3.6 Optimization Problems

. **Free Response** A right cone has a slant height of 6, as shown in the figure.



(a) Write the volume V of the cone as a function of one variable, r. The formula for the volume of a cone is

$$V = \frac{1}{3}\pi r^2 h.$$

(b) What are the dimensions that maximize the volume of the cone?

3.7 Differentials

Consider a function f that is differentiable at c. The equation for the tangent line at the point $(c, f(c))$ is

$$y - f(c) = f'(c)(x - c) \quad \text{or} \quad y = f(c) + f'(c)(x - c)$$

This is called the **tangent line approximation** (or **linear approximation**) **of** $\boldsymbol{f}$ at c.

Example 1: Find the tangent line approximation of $f(x) = 1 + \sin x$ at the point $(0,1)$. Then use a table to compare the y-values of the linear function with those of $f(x)$ on an open interval containing $x = 0$.

x	-0.5	-0.1	0.01	0	0.001	0.1	0.5
$f(x) = 1 + \sin x$							

3.7 Differentials

Definition of Differentials

Let $y = f(x)$ represent a function that is differentiable on an open interval containing x. The **differential of** $\boldsymbol{x}$ (denoted by dx) is any nonzero real number. The **differential of** $\boldsymbol{y}$ (denoted by dy) is

$$dy = f'(x)dx.$$

Example 2: Let $y = x^2$. Find dy when $x = 1$ and $dx = 0.01$. Compare this value with Δy for $x = 1$ and $\Delta x = 0.01$.

3.7 Differentials

Example 3: The measured radius of a ball bearing is 0.7 inch, as shown in the image. The measurement is correct to within 0.01 inch. Estimate the propagated error in the volume V of the ball bearing.

3.7 Differentials

Differential Formulas

Let u and v be differentiable functions of x.

Constant multiple: $d[cu] = c\,du$

Sum or difference: $d[u \pm v] = du \pm dv$

Product: $d[uv] = u\,dv + v\,du$

Quotient: $d\left[\frac{u}{v}\right] = \frac{v\,du - u\,dv}{v^2}$

Example 4: Complete the table

Function	Derivative	Differential
a. $y = x^2$		
b. $y = \sqrt{x}$		
c. $y = 2\sin x$		
d. $y = xe^x$		
e. $y = \frac{1}{x}$		

3.7 Differentials

Example 5: Find the following differentials.

a. $y = \sin 3x$

b. $y = f(x) = (x^2 + 1)^{1/2}$

3.7 Differentials

Example 6: Use differentials to approximate $\sqrt{16.5}$